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Research on Stock Price Volatility Based on GARCH-type Models

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Abstract:

This study performs an empirical investigation into the price volatility characteristics of China's stock market utilizing GARCH-family methodologies. Daily data of the Shanghai Composite Index from 2010 to 2024 are employed, and an ARMA-GARCH-type framework is constructed to capture the volatility structure of the return series. The analysis begins with stationarity and ARCH effect tests on the original series. The results indicate significant volatility clustering and conditional heteroskedasticity, justifying the application of GARCH modeling. Subsequently, alternative specifications are estimated, incorporating diverse distributional assumptions for the residuals, to assess their comparative goodness-of-fit. The results show that the EGARCH(1,1) model under the skewed t-distribution provides the best fit, effectively capturing both the asymmetric and heavy-tailed features of the return volatility. Ljung-Box and ARCH-LM tests on the standardized residuals confirm the adequacy of the model fit. The empirical findings suggest that volatility in China's stock market returns exhibits strong persistence and heightened sensitivity to negative shocks. These results offer quantitative insights for investor risk management and regulatory market surveillance, and also provide a methodological foundation for future research on multivariate volatility modelling.

Keywords: GARCH models, Stock price volatility, Conditional heteroskedasticity, EGARCH, Market risk

1.Introduction

In recent years, the global macroeconomic environment has been under sustained pressure, compounded by multiple adverse factors such as the COVID-19 pandemic and geopolitical conflicts. China's economic development faces significant uncertainty challenges, with increasing downward pressure on growth. As a "barometer" of the macroeconomy, stock market trends directly reflect the overall economic conditions and their cyclical fluctuations[1].. With the continuous expansion of China's stock market capitalization and the ongoing improvement of related infrastructure, its importance within the socio-economic system has become increasingly prominent, alongside a marked enhancement in its sensitivity to macroeconomic changes. Therefore, an in-depth study of the volatility patterns in the stock market is not only crucial for maintaining market stability and healthy development but also provides a vital practical perspective for better understanding macroeconomic dynamics[2].

Stock price volatility, as a core indicator of financial risk, has wide applications in financial practice, including asset pricing, portfolio optimization, and risk management. It also serves as a key tool for financial regulators to monitor systemic risks, aiding in the identification of potential financial vulnerabilities and the formulation of proactive policy measures[3]. Particularly in the current context of heightened global uncertainty and significantly increased market volatility, comprehensive research on stock price volatility contributes not only to advancing the theoretical understanding of the micro-mechanisms underlying financial markets but also holds practical significance for enhancing the risk resilience and early warning capabilities of China's financial system[4].

In view of this, the present study intends to employ the widely used GARCH-family models in financial econometrics to model and analyze the major stock market price indices in China.By estimating and fitting the model parameters, the study aims to reveal the overall trend and dynamic evolution patterns of stock market volatility in China. This research aspires to provide methodological support for market participants in risk identification and the construction of scientifically grounded investment decision frameworks, while also offering financial regulatory agencies empirically based and more precise volatility measurement tools, thereby improving the scientific rigor and targeted effectiveness of market risk monitoring and management.

2. Overview of GARCH-family Models

Stock price series typically exhibit several stylized facts, such as conditional heteroskedasticity, volatility clustering, and asymmetric responses to shocks. Traditional linear time series models often fail to accurately capture these dynamic structures. To address this limitation, Engle (1982) introduced ARCH model[5], which was later generalized by Bollerslev (1986) into the GARCH model, providing an effective theoretical framework for modeling the volatility of financial assets[6].

GARCH-type models construct dynamic equations for the conditional variance of return series, thereby capturing the persistence, time-varying nature, and potential structural characteristics of volatility. This modeling framework is also highly flexible and can be extended to accommodate empirical features such as asymmetry, long memory, and leverage effects. Consequentlyempirical investigations into financial market volatility routinely employ GARCH-class methodologies.

3. Empirical study

3.1 Data Selection and Sample Description

To investigate the volatility patterns of China's stock market, this study selects the Shanghai Composite Index as the research subject to construct the time series sample. The analysis employs 3,626 daily valid observations, with temporal coverage extending from 2010-01-04 through 2024-12-06.The Shanghai Composite Index is chosen because it comprehensively reflects the overall situation of China's stock market in terms of industry coverage and compilation principles. The data used in this paper are sourced from the Wind database. First, the price trend of the Shanghai Composite Price Index over the sample period is presented, as shown in Figure 1:

Figure 1. Price Trend of the Shanghai Composite Index

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As shown in Figure 1, within the sample period, the Shanghai Composite Index exhibits high frequency and large amplitude fluctuations. Around 2016, it experienced significant volatility characterized by a pronounced peak and a deep trough. Additionally, around 2020, the index underwent two substantial upward and downward movements, primarily due to the impact of the COVID-19 pandemic.

As noted earlier, the Shanghai Composite Index shows an obvious trending pattern over the sample period, so it is regarded as a non - stationary time series. Such time series are liable to spurious regression, which could make model results fail to fully reflect the authentic information of economic variables.Therefore, ensuring stationarity is generally a prerequisite before modeling financial time series. Accordingly, this study applies logarithmic differencing to the price series to obtain the return series, denoted as SZ. The calculation formula is as follows:

$$R_t = lnP_t - lnP_{t-1}$$

Where P_t and P_{t-1} represent the closing prices at times t and t−1, respectively, and R_t denotes the return at time t.

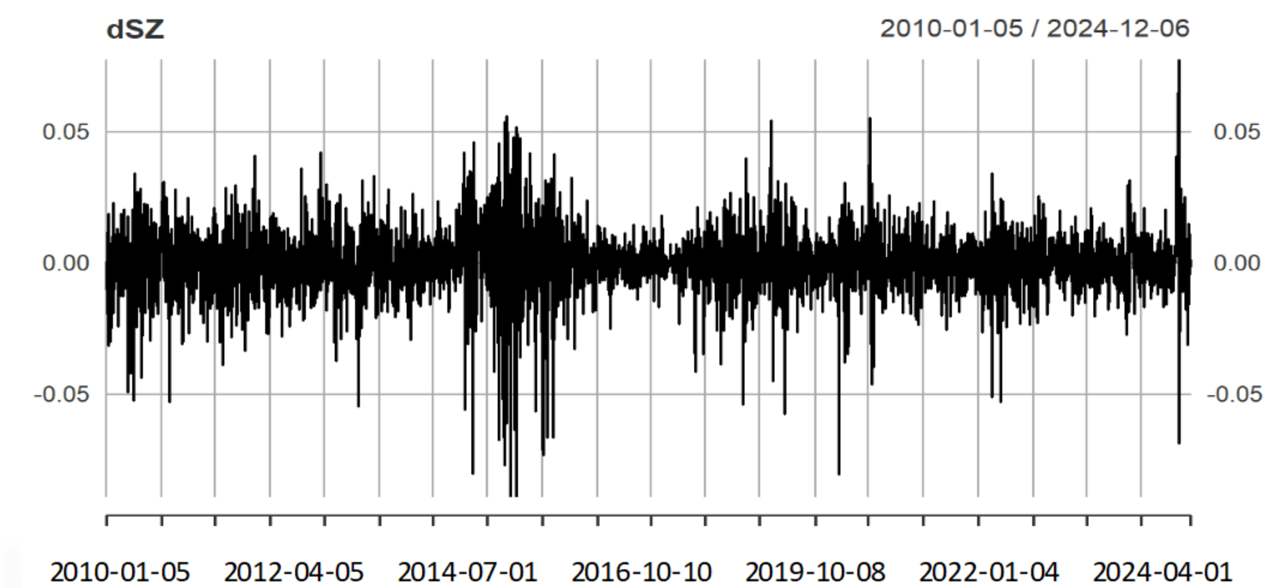

Figure 2. Logarithmic Return Trend of the Shanghai Composite Index

As shown in Figure 2, within the sample period, the logarithmic returns of the Shanghai Composite Index exhibit large amplitude and high-frequency fluctuations, along with instances of extreme volatility. Moreover, the return series displays a pronounced volatility clustering effect, where large fluctuations tend to be followed by large fluctuations, and small fluctuations tend to be followed by small fluctuations. This phenomenon is primarily attributed to the presence of autocorrelation and conditional heteroskedasticity.

3.2 Descriptive Statistical Analysis

Using the R programming language, this study provides the basic descriptive statistical analysis of the return series. The results are presented in Table 1:

Table 1. Descriptive Statistical Analysis of the Logarithmic Returns of the Shanghai Composite Index

nobs	Minimum	Maximum	Mean	Stdev	Skewness	Kurtosis	jarque-bera
3625	-0.0887	0.0776	0.0000	0.0128	-0.8039	6.6531	7087.1***

As shown in Table 1, the return series exhibits negative skewness, indicating a certain degree of left skewness. The kurtosis stands at 6.6531. Notably, it far exceeds the kurtosis value of 3 typical of a normal distribution. This signals that the return series deviates from a normal distribution, exhibiting marked leptokurtic (fat - tailed) traits. Further, the Jarque-Bera normality test yields a test statistic of 7087.1 with a p-value less than 0.01. At the 99% significance level, rejection of the null hypothesis (which assumes normality) occurs. It thus verifies that the series fails to conform to a normal distribution. Q-Q plot also demonstrates that the return series deviates from normality.

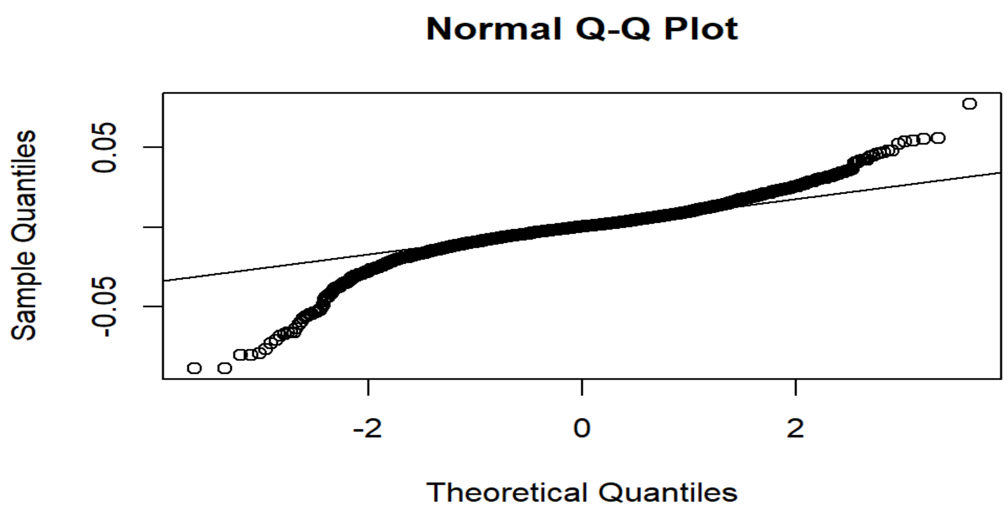

Figure 3. Q-Q Plot of the Logarithmic Returns of the Shanghai Composite Index

3.3 Stationarity Test

As previously noted, non-stationary time series are prone to spurious regression; therefore, before modeling the return series, it is necessary to verify its stationarity. This study employs the commonly used ADF and PP tests to assess the stationarity of the series. For both tests, the null hypothesis states that the series exhibits non - stationarity. Should the absolute value of the test statistic be sufficiently large and the p - value small, this null hypothesis can be rejected at a specified significance level, signifying that the series is stationary. Using the R programming language, the ADF and PP test results are presented in Table 2:

Table 2. ADF Test Results for the Logarithmic Returns

Test type	Test statistic value	P-value
ADF	-14.881	0.01
PP	-3522.2	0.01

The ADF test statistic is -13.924 with a p-value less than 0.01. At the 99% significance level, the null hypothesis of non-stationarity can be rejected, indicating that the return series is stationary. Similarly, the PP test statistic has a large absolute value and a p-value less than 0.01, allowing rejection of the null hypothesis of non-stationarity at the 99% significance level. Therefore, overall, the return series can be considered stationary and is suitable for further modeling and analysis.

3.4 Pre-estimation Diagnostics

Due to the pronounced volatility clustering effect observed in the return series, this study intends to employ GARCH-family models to characterize its volatility dynamics. Prior to modeling the return series with a GARCH model, it is necessary to conduct Ljung-Box and ARCH-LM tests to verify whether the series meets the prerequisites for applying GARCH models.

Table 3. Results of the Ljung-Box and ARCH-LM Tests for the Returns of the SSE 50 Stock Index Futures

	Ljung-Box Test		ARCH-LM Test	
	Test Statistic	P-value	Test statistic	P-value
Rt	33.612	0.0007	468.39	0.0000

As shown in Table 3, with a lag order of 10, the Ljung-Box test statistic for the return series is 33.612, with a p-value of 0.0007, which is less than 0.1. At the 90% significance level, the null hypothesis of no autocorrelation can be rejected. The ARCH-LM test statistic is 468.39, with a p-value less than 0.01, allowing rejection of the null hypothesis of no ARCH effect at the 99% significance level. Therefore, overall, the return series exhibits both

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autocorrelation and heteroscedasticity, satisfying the prerequisites for applying the GARCH model. Consequently, a GARCH model can be established to measure its volatility characteristics.

3.5 Construction and Analysis of GARCH-family Models

3.5.1 Mean Equation Fitting

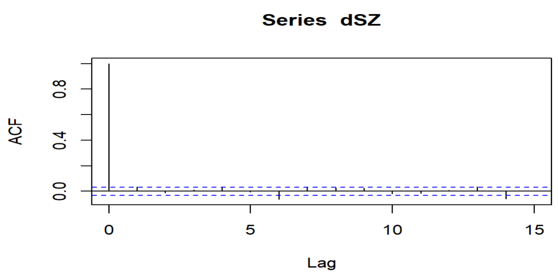

Figure 4. ACF Plot

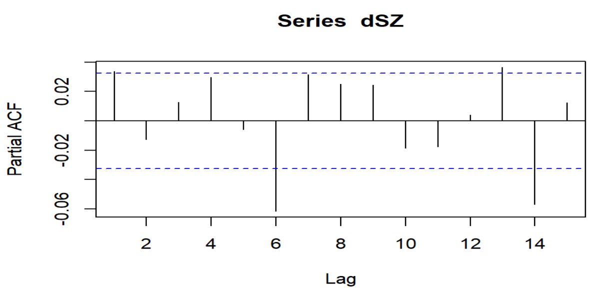

Figure 5. PACF Plot

Figures 4 and 5 respectively present the ACF and PACF plots of the logarithmic return series of the Shanghai Composite Index with a lag of 15 periods. From these plots, it is evident that both the ACF and PACF exhibit cutoff at lag 1, with values reaching the critical confidence interval at the first lag. This confirms that the return series exhibits a certain degree of autocorrelation and partial autocorrelation within the sample period. Considering the presence of autocorrelation in the logarithmic return series of the Shanghai Composite Index, this study adopts an ARMA model with a lag order of 1 as the mean equation for parameter estimation within the GARCH modeling framework.

3.5.2 Fitting of GARCH-family Models

According to relevant literature on GARCH models, the majority of studies adopt first-order lag models and the GARCH(1,1) specification to reduce the number of parameters and thus avoid compromising the accuracy of parameter estimation. Therefore, considering both the significance of parameter estimates and the adverse impact of excessive parameters on estimation accuracy, this paper employs an ARMA(1,1) model as the mean equation and estimates the GARCH(1,1), TGARCH(1,1), and EGARCH(1,1) models to characterize the volatility features of the Shanghai Composite Index during the sample period.

To further ensure the objectivity of the empirical process, three different conditional distributions—normal, t-distribution, and skewed t-distribution—are used to fit the GARCH(1,1), TGARCH(1,1), and EGARCH(1,1) models simultaneously. The AIC is then applied to select the optimal conditional distribution under which the GARCH-family model best fits the return volatility.

To determine the optimal model specification, this study compares the AIC values across different model specifications and residual distribution assumptions. The model and residual distribution assumption with the lowest AIC are selected as the optimal form, upon which parameter estimation and subsequent analysis are based. Using the R programming language, the AIC results for different model specifications and residual distribution assumptions are presented in Table 5:

Table 4. Maximum Likelihood Estimation Results of Each Model

	Distribution	AIC
GARCH(1,1)	norm	-6.1533
	t	-6.2428
	Skewed t-distribution	-6.2439
EGARCH(1,1)	norm	-6.1506
	t	-6.2528
	Skewed t-distribution	-6.2538
TGARCH(1,1)	norm	-6.1471
	t	-6.2408
	Skewed t-distribution	-6.2420

As shown in Table 5, among the GARCH-family models, the EGARCH model with the conditional distribution assumption of a skewed t-distribution yields the lowest AIC value of -6.2538. Therefore, this study adopts the EGARCH(1,1) model with a skewed t-distribution assumption to fit and measure the volatility characteristics of the Shanghai Composite Index during the sample period.

4. Research Conclusions

4.1 Estimation Results

After selecting the optimal model specification and residual distribution assumption, this study employs maximum likelihood estimation to estimate the parameters of the specified EGARCH(1,1) model. Furthermore, given that daily data are used, and considering the timeliness and effectiveness of information transmission, the mean equation is assumed to follow a moving average process. To determine the optimal ARMA lag order, this study jointly estimates the EGARCH(1,1) volatility model with mean equations specified as the optimal lag-1 ARMA model and the ARMA(1,1) model fitted to the logarithmic returns of the Shanghai Composite Index. The detailed parameter estimation results are presented in the following table.

Table 5. Parameter Estimates of the ARMA(1,1)-EGARCH(1,1)-sstd Model for the Returns of the Shanghai Composite Index

	Estimate	Std. Error	T value	Pr(>|t|)
mu	0.000032	0.000161	0.19899	0.8423
ar1	-0.35655	0.049796	-7.16017	0.0000
ma1	0.354875	0.049765	7.13094	0.0000
omega	-0.11439	0.012549	-9.11526	0.0000
alpha1	0.059282	0.008032	7.38073	0.0000
beta1	0.933048	0.008045	115.9765	0.0000
gamma1	0.148607	0.025791	5.76204	0.0000
skew	0.949252	0.020229	46.92548	0.0000
shape	4.630462	0.313871	14.75275	0.0000

As shown in Table 6, in the mean equation, except for the constant term (mu), the coefficients of the autoregressive and moving average terms have large absolute values of their t-statistics and very small p-values, all significant at the 1% level. Therefore, at the 1% significance level, the null hypothesis of insignificance for these coefficients is rejected. Both AR(1) and MA(1) terms are significant, indicating that the mean of the Shanghai Composite Index returns overall follows an ARMA(1,1) process.

Within the conditional variance equation, the ARCH and GARCH components of the EGARCH model correspond respectively to the volatility and volatility clustering characteristics of the time series. Based on parameter estimation outcomes, the EGARCH model successfully captures the volatility clustering phenomenon in the logarithmic returns of the Shanghai Composite Index.The ARCH parameter (alpha1) has a p-value less than 0.05, indicating its significant sensitivity to market volatility and notable volatility characteristics. The GARCH parameter (beta1) suggests strong volatility clustering in the return series, implying pronounced long memory and persistence.

The sum of the ARCH and GARCH coefficients (alpha1 + beta1) is close to 1, indicating that changes in volatility itself have a persistent impact on the conditional variance. This suggests that the logarithmic returns of the Shanghai Composite Index tend to experience high volatility that persists over extended periods.

Furthermore, the asymmetric term coefficient is 0.1438607, with a corresponding p-value significantly less than 0.05, confirming a pronounced asymmetric effect in the Shanghai Composite Index. This asymmetry implies that the index is more sensitive to negative shocks.

Additionally, the estimates of the skewness and shape parameters are significant at the 5% level, indicating that the standardized residuals of the Shanghai Composite Index logarithmic returns exhibit clear asymmetry and heavy tails.

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4.2 Model Fit Evaluation

Table 6. Model Fit Evaluation

Test type	Test statistic
ARCH	5.8298 (0.9244)
LB	8.4424 (0.5856)

LB represents the Ljung-Box test on the standardized residuals; ARCH denotes the ARCH effect test on the standardized residuals; values in parentheses are the corresponding p-values of the test statistics."

After fitting the logarithmic returns of the Shanghai Composite Index with the EGARCH(1,1) model, the standardized residuals were tested for autocorrelation and ARCH effects to evaluate the model's fit. The Ljung-Box test returned a p-value of 0.5856, a figure exceeding 0.05. This result demonstrates that when using the 5% significance threshold, the standardized residuals from the fitted series exhibit no autocorrelation.Similarly, the ARCH-LM test produced a p-value of 0.9244, also greater than 0.05, demonstrating that no ARCH effect exists in the standardized residuals at the 5% significance level after fitting with the EGARCH(1,1) model.

These results confirm that the EGARCH(1,1) model fits well and can accurately characterize the distributional properties of the Shanghai Composite Index logarithmic return series. Therefore, the EGARCH(1,1) model with a skewed t-distribution will be used to estimate the volatility of the Shanghai Composite Index.

The QQ-Plot of the model residuals indicates that the residuals approximately follow a normal distribution. The QQ-Plot of the residuals is shown in Figure 3:

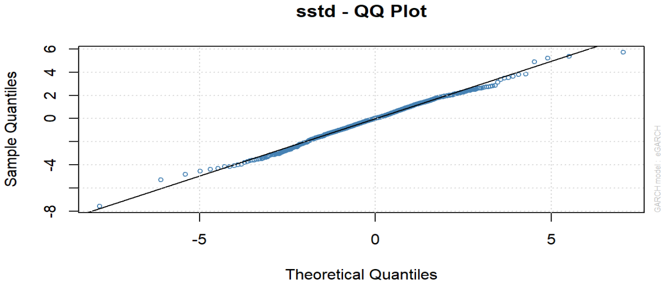

Figure 3. QQ-Plot of Model Residuals

Overall, the EGARCH(1,1)-sstd model constructed in this study effectively filters out the volatility clustering present in the original series, demonstrating a good model fit.

The volatility estimated by the conditional variance of the EGARCH(1,1) model is shown in Figure 4:

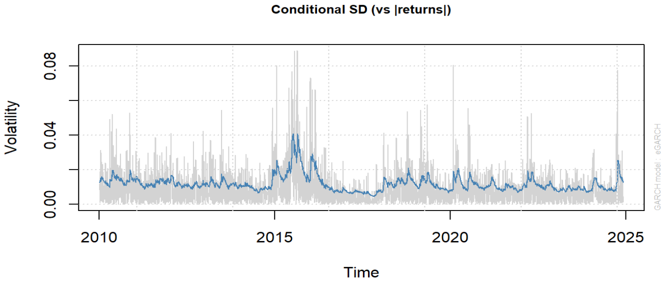

Figure 4. Volatility Trend of the Return Series

The volatility of the return series is generally high, primarily ranging between 0.01 and 0.04, with some degree of variation. Around 2016, volatility reached a trough and peak, which coincides with the stock market crash during that period, indicating significant fluctuations in China's stock market. Around 2020, the overall market volatility remained at a relatively high level, although no severe turbulence occurred. This relative stability can largely be attributed to China's effective pandemic prevention policies, which helped prevent sharp market fluctuations.

Overall, the volatility of the Shanghai Composite Index remained relatively stable during the sample period. Except for a few periods marked by abrupt changes, the volatility level was largely consistent throughout.

4.3 Policy Recommendations and Practical Implications

Based on the research findings, the following policy recommendations are proposed:

First, investors should pay close attention to the dynamic changes in market volatility and make rational use of volatility information to optimize asset allocation and risk management. An increase in volatility often signals heightened market uncertainty. In response, investors may consider adjusting their portfolio structure by increasing the proportion of defensive assets or employing hedging strategies through derivatives such as options. Given the market's heightened sensitivity to negative shocks, investors should remain rational in the face of adverse information and avoid emotionally driven decision-making.

Second, regulatory authorities should incorporate volatility measures based on GARCH-type models into the macroprudential regulatory framework for the dynamic monitoring of systemic risks. In the event of significant increases in volatility, timely issuance of risk warnings, enhanced market supervision, and the implementation of counter-cyclical regulatory measures—such as adjusting margin requirements and leverage limits—can help prevent financial risks from transmitting to the real economy. Furthermore, efforts should be made to strengthen the application of volatility modeling techniques in regulatory practices, thereby improving the scientific basis of risk identification and policy responses.

Finally, this study provides a practical example of applying GARCH-type models to characterize volatility behavior under the conditions of the Chinese market. Future research may extend the model framework by exploring multivariate GARCH structures and volatility spillover effects between markets, as well as incorporating high-frequency data to enhance predictive accuracy.

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