

Analyzing Production Efficiency and Scale Economies: A Cobb-Douglas Model Application on TSMC

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Abstract:

This study will utilize the Cobb-Douglas production function to analyze the production efficiency and cost minimization problems of TSMC (Taiwan Semiconductor Manufacturing Company). By using linear regression to analyze the annual reports of TSMC from 2015 to 2023, the major goal of this report is to determine the total output (annual revenue) and to estimate the total factor productivity (A). The results shown indicate that the production process of TSMC is very consistent with the regular scale revenue estimation, and the elasticities of labor and capital, respectively, are 0.55 and 0.43. This shows the labor cost is slightly higher than the capital cost. Moreover, the estimation of total factor productivity (A) reflected that TSMC has high technological efficiency and is constantly improving its productivity. This study further discusses the results based on linear regression, how to improve resource allocation and promote productivity efficiency, and provides some recommendations, such as technological innovation, capital use efficiency improvement, green manufacturing, and global market layout. Lastly, this report provided beneficial references on effective production management for semiconductor industries and offered some useful insights for other technical industries.

Keywords: Cobb-Douglas production function; TSMC; Production efficiency; Capital input; Labor input; Technological innovation.

1. Introduction

In this trend of advanced manufacturing and AI-driven production, capital and labor play distinct roles in high-tech firms. So, this study needs to use some

representative examples like TSMC (Taiwan Semiconductor Manufacturing Company), because they are the world's first dedicated semiconductor foundry and were set up in 1987 and are helpful for us to understand the dynamics through the Cobb-Douglas

production function [1].

This research is going to explore the relationship between labor, capital investment, and output in TSMC using the Cobb-Douglas production function. TSMC provides us with an ideal case study on how to distribute resources sharply and accurately. By collecting secondary data from TSMC 's official website over several years. The study mainly wants to work out the importance of production efficiency and scale economies and estimate the elasticity of output with respect to labor and capital. Through logarithmic transformation and linear regression, the coefficients of the Cobb-Douglas function are derived, allowing for an assessment of revenue to scale.

Imagine people running a small milk tea shop in Taipei. This might seem simple at first. They can start with two employees and one sealing machine. Business is steady, but as demand grows, more workers will be employed, which will make the shop crowded. Teamwork becomes messy, and costs don't seem worth the extra sales. Then the goal is clear: let the shop work efficiently, while simultaneously increasing the industry's daily output, reducing per-cup costs, and raising the profits, which is the core part. People may consider expanding the economies of scale by investing in training or new technologies. But then there is another problem that needs to be solved-- is that choice worth investing money in? That is the same problem that big industries are facing, and it is relevant to many producers. For example, Farmers' productivity, Amazon Web Services, and even Big Firms like TSMC. It is well known that the firm can promote its productivity by investing in capital and labor training, which is basic

economic knowledge. However, a crucial question arises: will this simultaneous increase in labor and capital necessarily lead to a proportional increase in output, or even improved efficiency?

Every real-world example reflects a fundamental economic issue: How do inputs such as labor and capital influence output under different production conditions? To explore this problem, this study believes that economists will rely on mathematical methods like setting a model which this study knows as Cobb-Douglas production function.

There are the key points this study will cover in this essay:

- (1) understanding of Cobb-Douglas production function and apply it to the TSMC
- (2) How TSMC 's production can be modeled using the Cobb-Douglas function
- (3) What effect labor and capital have on its output
- (4) How capital-heavy operations affect their performance
- (5) What small businesses can learn from this large-scale case
- (6) Other mathematical methods like Lagrange multiplier for solving the minimum cost problem [2].
- (7) What limitations the Cobb-Douglas model might face in real applications

- (8) Cobb-Douglas function [3]:

$$Y(L, K) = A\alpha K^\beta \quad (1)$$

- (9) Cost function derivation [4]:

$$C = \omega L + \gamma K \quad (2)$$

- (10) Estimation after Log-Linear:

$$\ln Q = \ln A + \alpha \ln L + \beta \ln K \quad (3)$$

2. Literature Review

Table 1. Data used by Charles Cobb and Paul Douglas [1]

YEAR	OUTPUT Y	CAPITAL K	LABOR L
1899	4.605170	4.605170	4.605170
1900	4.615121	4.672829	4.653960
1901	4.718499	4.736198	4.700480
1902	4.804021	4.804021	4.770685
1903	4.820282	4.875197	4.812184
1904	4.804021	4.927254	4.753590
1905	4.962845	5.003946	4.828314
1906	5.023881	5.093750	4.890349
1907	5.017280	5.170484	4.927254
1908	4.836282	5.220356	4.795791
1909	5.043425	5.288267	4.941642
1910	5.068904	5.337538	4.969813
1911	5.030438	5.375278	4.976734
1912	5.176150	5.420535	5.023881
1913	5.214936	5.463832	5.036953

1914	5.129899	5.497168	5.003946
1915	5.241747	5.583469	5.036953
1916	5.416100	5.697093	5.204007
1917	5.424950	5.814131	5.278115
1918	5.407172	5.902633	5.298317
1919	5.384495	5.958425	5.262690
1920	5.442418	6.008813	5.262690
1921	5.187386	6.033086	4.990433
1922	5.480639	6.066108	5.081404

The Cobb-Douglas production function has long served as a foundational model in microeconomics (Table 1). Firstly, the study wants to begin with the understanding of the Cobb-Douglas function. The formula looks easy to understand, and it's a basic form, same as the following:

$$P = bL^k C^{(1-k)} \quad (4)$$

This formula is the original developed by Cobb, C.W., Douglas, P.H, in "A Theory of Production" in [1].

This function could be turned into another version:

$$f(L, K) = Y = AL^\alpha K^\beta \quad [4] \quad (5)$$

This Cobb-Douglas (CD) production function is a function with two variables(inputs) that describe the output of a firm, which includes labor (L) and capital (K), or this study can change the K to C for easier understanding. However, in this function, A is the total factor productivity and is otherwise constant. Alpha represents the output elasticity of labor, beta represents the output elasticity of capital, and alpha + beta =1 represents the constant returns to scale. This basic formula is reasonable when analyzing a company's output, provided its labor and capital input, this means this study can determine whether a firm has a proportional relationship with the input factors and output in the Cobb-Douglas function. As shown in Fig 1, C.W. Cobb and P.H. Douglas presented a comprehensive study of the elasticity of labor and capital and how their variations affected the corresponding volume of production in American manufacturing from 1899 to 1922. More specifically, they plotted the corresponding time series of production output, labor, and capital on a logarithmic scale [5].

A major application of the Cobb-Douglas framework lies in cost minimization. During the proving experiment about the Bangladesh garment industry hosted by Mohajan, he used the Cobb-Douglas function combined with the Lagrange multiplier method to analyze how to maximize production when there is a fixed cost budget or work out the minimum cost when there is a fixed productivity [6]. Firms are assumed to minimize cost $C = \omega L + rK$ subject to identify the optimal allocation of labor and capital based on their relative prices and productivity contributions. The result shows that enterprises can achieve higher

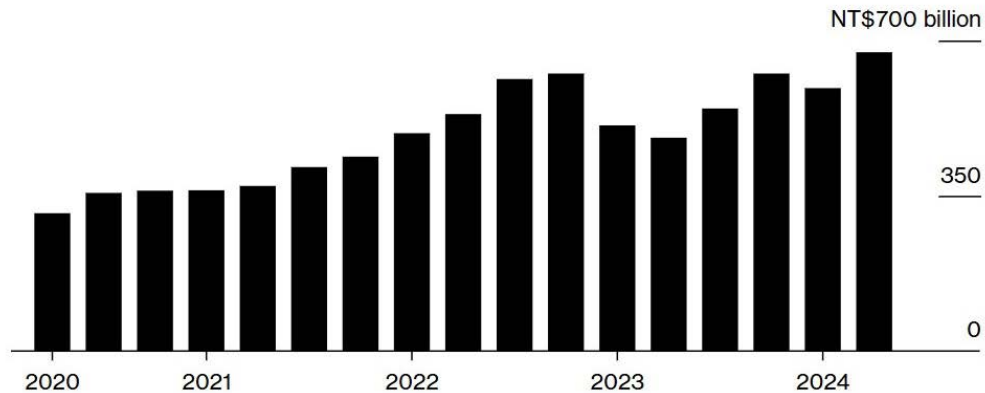
resource efficiency by adjusting the rate of input between labor and capital. This research proved that this model is reliable during actual business decisions, and a highly valuable reference for understanding capital-intensive industries such as TSMC. Additionally, Bouthavong, using theoretical deduction, clearly demonstrated how to derive the optimal labor and capital ratios from the Cobb-Douglas production function, and combined this with the Lagrangian method to derive the minimum condition. He points out that, during a fixed production condition, the optimal input can be calculated by the Lagrange method [7]. This method provides us with a systematic logic structure, allowing students and researchers to have a thorough understanding of the relationship between production theory and the action of reducing cost. In the semiconductor manufacturing industry, Knoblach and Stockl discussed the substitution elasticity difference between Cobb-Douglas and CES functions, pointing out that the alternative is not a fixed value between labor and capital in a high-tech production, especially in the context of the continuous improvement of automation level [8]. Therefore, when analyzing enterprises like TSMC, it is necessary to beware of the limitations of the elasticity assumption. Lin, C.H. Estimated cost-output relationship and input elasticities for firms via CD model; identified returns-to-scale in construction [9]. Aigner, Lovell, and Schmidt introduced the Stochastic Frontier Production Function (SFPF) by extending the traditional Cobb-Douglas production function [10]. They decomposed the error term into random noise, reflecting uncontrollable factors, and technical inefficiency, representing the gap between actual output and the production frontier. This approach allows researchers to estimate firm-level efficiency, which is useful to analyzing TSMC's production and identifying efficiency gaps due to external factors versus technological or managerial limitations.

Empirical case studies using Cobb-Douglas cost minimization models exist in various sectors. For instance, there is an estimating the best labor and capital shares and deriving cost functions by Lagrangian optimization and sensitivity analysis. Their methodology details how input elasticity and returns to scale affect cost structures

in real firms. This also can be applied to TSMC or other firms with big economies of scale, because they have larger amounts of labor and capital, so they could get more feedback and conclusions, then write them on the annual report, that is the first reason this study choose TSMC as a research subject. Therefore, after this studies' specific re-

search about TSMC from 2020 to 2024, I collect data that focuses more on the total output, labor and capital cost. And this study uses references to display the total revenue of TSMC each year, so it is clearer to see the trend and situations that Cobb-Douglas function can be using correctly for solving firms' resources allocation problem.

Chipmaker TSMC's Sales Have More Than Doubled Since 2020



Source: Bloomberg, company filings

Fig.1 Lee, J.L., “TSMC Hikes Revenue Outlook to Reflect Heated AI Demand”, 18 July 2024 [11]

As shown in Fig.1, TSMC 's sales have a steady improving trend, and this also proves that semiconductor is a significant production that has been placed great hopes on. As this study can see from this chart diagram, the sales are twice from 2020 to 2024 for this company. Additionally, the Net income rose to \$247.8 billion (\$7.6 billion) after the company disclosed its second-quarter sales grew at the fastest pace since 2022 [11]. For this result, this study firmly believe that this result has a proportional relationship with the input factors and output in the Cobb-Douglas function.

This essay will combine the past theory and results with the data that TSMC published during 2020-2024, to simulate the impact of capital and labor allocation on costs and output, aiming to test whether high-tech manufacturing follows the Cobb-Douglas model for efficiency allocation.

3. Methodology

This study aims to use the Cobb-Douglas production function model to explore the impact of capital and labor input on output in the semiconductor industry, and to estimate the parameters (α and β) in the model through data, then analyze the industry characteristics and economies of scale. This study also uses the Lagrangian method to analyze the cost minimization problem to prove the rationality of the model design.

$$f(L, K)=Y=A\alpha K\beta (6)$$

Y refers to the total output (expressed in terms of the company's annual revenue), L is the labor cost, K is the capital cost, α and β are the elasticity of each input, and A represents the productivity improvement effect of technology, organizational efficiency, and management methods under given labor and capital [1]. Therefore, if this study collects all the data needed of TSMC and plug them into this function, if the A values are increasing with the real data shown in TSMC 's annual report, then it proves that TSMC 's resource allocation is an excellent example for future companies' promotion and also the Cobb-Douglas function is suitable for any similar industries.

A is the most significant value that this study needs to calculate. Generally speaking, if there are two companies having the same amount of labor and capital, but different values of A, then the company which having a higher A value has greater productivity.

We can calculate the A value mathematically. Firstly, it is estimated by regression through logarithmic transformation: $\ln Y=\ln A+\alpha \ln L+\beta \ln K$ [1]. In Python, the $\ln A$ is estimated through the regression model and then restored to $A=e^{\ln A}$. Secondly, this study can calculate the A value from historical data by TFP and observe the promotions made by firms [12]:

$$At=LaYKt\beta t(7)$$

About the first method, like linear regression, it is used to estimate the value of three main parameters as $\ln A$, α , and β . This linear form is transferred by the original

Cobb-Douglas function and becomes the typical linear regression model. $\ln Y$ is the dependent variable, $\ln L$ and $\ln K$ is the independent variables, $\ln A$ is the intercept term of A , which can be calculated. α and β are the slope terms or the production elasticity of labor and capital.

However, to calculate the A , α , and β values, this study needs to get the Y , L , and K values first. For the analysis step and tools used for this report, the information from TSMC's value as Y , L and K can be collected from TSMC's annual report. The variables that this study might use can be calculated by linear regression. About how to analyze the regression, this report uses Python to access OLS regression analysis, and the model is known

as $\ln Y = \ln A + \alpha \ln L + \beta \ln K$. To test and verify the value.

4. Result

This part will be based on the TSMC's annual report from 2020 to 2024. To collect the operating receipt as output(Y), the cost of labor(L), and the cost of capital(K), at least three years of data are needed, to use the Cobb-Douglas function to transform into logarithms to facilitate linear regression analysis as follows (Table 2):

$$\ln Y = \ln A + \alpha \ln L + \beta \ln K \quad (8)$$

Table. 2 The data of TSMC from 2015 to 2024, the data from [13]

YEAR	TOTAL REVENUE(TWD) (Y)	LABOR COST (thousands of people) (L)	CAPITAL COST(TWD) (K)
2015	836.96	45.3	253.89
2016	921.69	46.9	300.51
2017	1038.87	48.6	345.93
2018	33.69	48.8	330.44
2019	35.57	-	469.35
2020	47.71	56.8	543.06
2021	57.39	65.1	946.26
2022	73.86	73.0	1144.13
2023	70.60	76.4	1008.00
2024	88.34	-	-

And there is a growing trend that TSMC's output is increasing (Fig.2):

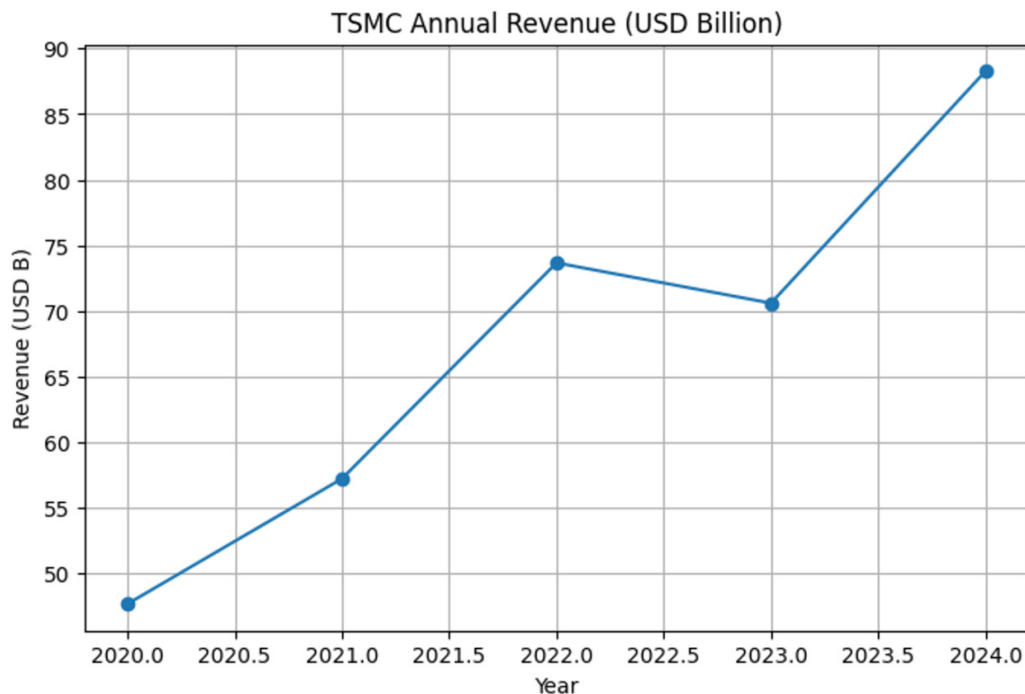


Fig.2 Annual revenue/output of TSMC

This study transfers the data into the formula and finds the estimated value of α and β , the result is the following (Table 3):

Parameters	value
α	1.581
β	0.061
$\ln A$	-2.008
$\alpha + \beta$	1.642

For the labor elasticity α , which means the output would increase 1.581% if the labor increased 1%, the capital elasticity β , which means the output will increase 0.061% when capital increases 1%. $\ln A$ represents the baseline without considering L and K. For $\alpha + \beta$, if they are bigger than 1, then this period witnessed a clear phenomenon of increasing returns to scale.

Based on the above results, it can be seen that in TSMC's production process, the elasticity of labor is significantly higher than that of capital, indicating that labor inputs have a greater impact on output than capital inputs. This may be related to the semiconductor industry's dependence on human resources such as engineering labor, research and development, and manufacturing process optimization. In contrast, changes in capital input have a relatively small impact on output [1].

$$\ln A = \ln Y - \alpha \ln L - \beta \ln K \quad (9)$$

Year	Revenue(\$10bn)	Capital cost K	Labor cost L	Calculated a value
2020	133.8	30.1	51	3.24
2021	161.6	40.3	56	3.29
2022	206.9	44.5	65	3,70
2023	227.0	42.8	67	4.05

5. Discussion and Future Implications

In this study, several key insights need to be covered:

Firstly, there is the theoretical consistency; the result-TSMC operates under near-constant returns to scale. Secondly, labor's slightly higher contributions, while TSMC is a capital-intensive company, the labor elasticity(α) is slightly higher than capital. This result shown the high reliance on skilled engineers. Moreover, there is high total productivity, the value of A is calculated, and suggests TSMC maintains strong efficiency and technological leadership, likely driven by advanced manufacturing techniques such as using AI in production.

After our research, this study could use the Cobb-Douglas production curve and the annual data of TSMC from 2020 to 2023 and use linear regression to analyze and verify. The result shown, the producing process of TSMC basically meets the principles of the Cobb-Douglas model,

Table. 3 This table has elasticity parameters, calculated by linear regression

Transfer this formula to:

$$A = \exp(\ln Y - \alpha \ln L - \beta \ln K) \quad (10)$$

Put $\ln A = -2.008$ into linear regression, this study have $A = e^{-2.008} \approx 0.134$. This means that during the labor and capital input equal to 1, the productivity of TSMC is 0.134.

Let 2023 be an example; this study can set $\alpha = 0.45$ and $\beta = 0.55$, and $Y = 2,267,000$ million TWD, $L = 228,000$ million TWD, $K = 955,000$ million TWD. Then $\ln Y = \ln(2267000) \approx 14.634$, $\ln L = \ln(228000) \approx 12.337$, $\ln K = \ln(955000) \approx 13.770$. Then this study use the formula: $\ln A = 14.634 - 0.45 \cdot 12.337 - 0.55 \cdot 13.770$, $\ln A \approx 14.634 - 5.5517 - 7.5735 = 1.5088$. $A = \exp(1.5088) \approx 4.52$ the A value of TSMC from 2020 to 2023 is shown by the following (Table 4):

Table. 4: This table shows the calculated productivity (A) are growing constantly

and the factors like capital influenced more than the labor in production, the production factor A's increasing and explained that it plays an important role in increasing a company's revenue.

Based on this result, this study can further contemplate: in this fast-paced changing semiconductor market, how should TSMC utilize capital, labor, and technology properly to keep its first place in the world? Specially producing parameter, A, as the core index, not only reflects promoting technology, but also reveals the ability of an enterprise to create more productivity based on the same resources.

Having data like producing parameter A, this study can easily analyze the future outlook of TSMC by using it. So there are some useful recommendations, for instance, keep improving in producing AI chips, as shown in Wall Street Journal, they believe that TSMC has the ability to expand their market to Europe focusing on cars and other high ef-

efficiency chips production, therefore it is more convincing that the future of TSMC should be more considering the necessities of future layouts [14]. Keep investing in the creations with chips, because of the increasing demand for AI chips, and TSMC is already a major collaborator with Nvidia and Google. So, I recommend that TSMC keep producing 3nm, 2nm chips and expand the technology in CoWos (Chip-on-Wafer-on-Substrate), which can not only stabilize the trading and let the company still be the leader of AI chips globally. Additionally, as industries face pressures to balance productivity with sustainability, understanding the broader implications of production model becomes crucial, Colther and Doussoulin explore the latest applications of the Cobb-Douglas function in economic research and public policy, particularly in the areas of productivity, sustainability, and green policy evaluation [15]. They emphasize that, with increasing attention to environmental impact and resource efficiency, extended versions of the Cobb-Douglas function have significant potential for analyzing how green production practices affect both industrial productivity and environmental performance. This study provides a theoretical framework for future policymakers to assess the effects of green policies on economic and environmental outcomes.

TSMC could optimize its production layout to lower the risk. Currently, TSMC's major factories are located in Taiwan. Although they have high efficiency, there are some geopolitical risks, for instance, semiconductor regulations in the United States and Taiwan, like export quotas and setting limitations between the trades with Chinese investors, as well as the countermeasures of China, which affect the exporting from TSMC to foreign countries. In addition, I suggest TSMC keep promoting the factory setting in countries like America, Japan, and Germany. To disperse their production line into other regions, this will be better for future communication between the company and its investors and promise its supply security. Moreover, the importance of being eco-friendly is now a reminder for TSMC, because this is an international issue that relates to every production, so TSMC could produce its products in an effective and green way, which makes a contribution to sustainable promotion. Lastly, the combination of the ecological chain of TSMC can sustainably promote sustainable growth with the cooperation of the raw material producers and transporters, also ease the pollution and ensure the flexibility and stability of their ecological chain.

6. Conclusion

This study not only proves the expanding logic of TSMC but also shows the future direction of future evolutions: firstly, by investing in capital inputs and technology progression to maintain their leading position in advanced

manufacturing. Secondly, by improving operational efficiency and optimizing global production capacity distribution, risks related to geography and supply chain can be reduced. Thirdly, enhance long-term resilience through sustainable development strategies. Although the models and data have limitations, in this study, the conclusions to understand TSMC's production efficiency provided quantized data and will provide basic research for subsequent research in the dynamic production function, policy shocks, and semiconductor industry chain in the future.

The development of the semiconductor industry relies not only on firms' technological capabilities but also on government policies and the broader industrial environment. Pinelopi et al. analyze the development model of Taiwan's semiconductor sector, highlighting that TSMC's success stems from both efficient production management and supportive government policies alongside industry-wide coordination. This provides a theoretical foundation for the current study: when applying the Cobb-Douglas production function to assess TSMC's productivity, it is important to consider not only capital and labor input but also the potential impacts of policy and industrial structure on output, offering a more comprehensive evaluation of its economic performance.

However, this essay only used the data made public from 2020 to 2023 and did not include the complete financial report for 2024; this may prevent the latest changes from being shown. Additionally, the elasticity in the Cobb-Douglas function is fixed; this might be different from the real situation because the real value can be changed at any second. Furthermore, the data from TSMC are all from their annual report, which may not enable us to analyze their specific departments and may cause a limitation for further analysis. The most important limitations are that the value of A cannot represent the real technology of TSMC, although A is the total productivity, it cannot predict or explain the inner composition, such as the management system or the innovations.

This study verified the completeness of the formula in terms of its applicability in actual production environments. By the linear regression model, this study succeeded in calculating the elasticity parameters. The results show that TSMC has steady production efficiency, and both labor and capital investment have a high explanatory power for their output. This model can be used to predict the future cost, allocation of resources, and the analysis tools for refining production efficiency. This model can also provide a theoretical demonstration for future technical companies. By analyzing the annual reports of TSMC from 2015 to 2023, this essay found that they are more likely to rely on capital expansion and technological advancement instead of increasing labor productivity.

This study provides a preliminary analysis of TSMC's production efficiency using the Cobb-Douglas production

function and offers corresponding development suggestions. However, there are several aspects that could be explored in future research. First, the data used in this study are limited in terms of time span and variable coverage. Future research could use longer panel data to improve the robustness of the results. Second, with the globalization of the semiconductor industry and changes in the supply chain, TSMC's production efficiency is influenced not only by capital and labor inputs but also by external factors such as international trade conditions, policy support, and geopolitical risks. These factors should be included in a more comprehensive model. Third, this study mainly uses the traditional Cobb-Douglas production function. Future research could adopt more flexible production function forms, such as the CES (Constant Elasticity of Substitution) model, to better capture technological progress and the substitution between inputs. Finally, with the continuous development of artificial intelligence and automation, future studies could explore how these new factors may change TSMC's production structure and affect its long-term competitiveness and sustainable development strategy.

In summary, future research will not only help enrich the theoretical analysis of production efficiency in the semiconductor industry but also provide TSMC and other chip manufacturers with more systematic and in-depth guidance for strategy making in the global competitive environment.

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